OCR
Oxford Cambridge and RSA

Friday 22 June 2018 – Morning

A2 GCE MATHEMATICS (MEI)

4769/01 Statistics 4

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4769/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

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INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.

- The Question Paper will be found inside the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer any **three** questions.
- Do **not** write in the barcodes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

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- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **12** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER/INVIGILATOR

- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Option 1: Estimation

1 A biased coin has probability p that it gives a tail when it is tossed. It is known that $0 < p < 1$. The random variable T is the number of tosses up to and including the second tail.

(i) Show that, for $t \geqslant 2$, $\mathrm{P}(T = t) = (t-1)(1-p)^{t-2}p^2$. **[3]**

(ii) A single observation of T takes the value t. Find the maximum likelihood estimate for p based on this observation. You may assume that any stationary point that you locate is a maximum. **[4]**

(iii) Show that $Z = \dfrac{1}{T-1}$ is an unbiased estimator of p. **[5]**

An alternative method for finding an unbiased estimator of p is to consider the random variable F, the number of tosses up to and including the first tail.

(iv) Show that $\dfrac{1}{F}$ is not an unbiased estimator of p. You may use the fact that

$$\sum_{n=1}^{\infty} \frac{x^n}{n} = -\ln(1-x) \text{ for } |x| < 1.$$ **[4]**

(v) The random variable Y is defined by

$$Y = \begin{cases} 1 & \text{if } F = 1, \\ 0 & \text{otherwise.} \end{cases}$$

Show that Y is an unbiased estimator of p. **[3]**

You are given that, for the estimator Z defined in part **(iii)**, $\mathrm{Var}(Z) = p^2\left(\dfrac{1}{1-p}\ln\left(\dfrac{1}{p}\right) - 1\right)$.

(vi) Show that if $p = 0.5$ then $\mathrm{Var}(Y) > \mathrm{Var}(Z)$. State, with a reason, which of Y or Z is the more efficient estimator of p in this case. **[5]**

Option 2: Generating Functions

2 The random variable X has probability density function

$$\mathrm{f}(x) = \begin{cases} \dfrac{1}{2b}\mathrm{e}^{\frac{x}{b}} & \text{for } x \leqslant 0, \\ \dfrac{1}{2b}\mathrm{e}^{-\frac{x}{b}} & \text{for } x > 0, \end{cases}$$

where b is a positive parameter.

(i) Show that, for $-\dfrac{1}{b} < t < \dfrac{1}{b}$, the moment generating function $\mathrm{M}_X(t)$ of X is $\dfrac{1}{1-b^2t^2}$. **[9]**

(ii) Hence find the expectation and variance of X in terms of b. **[6]**

A random variable Y with moment generating function $\mathrm{M}_Y(t) = \dfrac{1}{1-bt}$ is said to have an exponential distribution with parameter b, denoted by $\mathrm{Exp}(b)$.

(iii) Two independent observations of Y are denoted by Y_1 and Y_2. Show that $Y_1 - Y_2$ has the same distribution as X. **[5]**

(iv) A radioactive substance emits beta particles. The time, in seconds, between successive emissions is called the 'waiting time' and has an Exp(4) distribution. Find the probability that the absolute value of the difference between two independent waiting times is more than 0.1 seconds. **[4]**

Option 3: Inference

3 A manufacturer produces large numbers of a particular type of power supply which is specified as delivering its output at 15 volts.

(a) It is important that the actual voltage should not be too high, as that could damage the equipment to which the power supply is attached. So the manufacturer's quality control department routinely tests each batch of power supplies. The null and alternative hypotheses under test are

$$H_0: \mu = 15.0, \qquad H_1: \mu > 15.0,$$

where μ volts is the true mean voltage delivered by the power supplies in the batch.

The test consists in drawing a random sample of size n from the batch, and comparing the mean voltage, $\bar{x}$, with a critical value c. The test is required to have a Type I error of size no greater than 0.05. The size of the Type II error, when $\mu = 15.1$, is to be no greater than 0.01.

Previous data indicate that the voltages are Normally distributed with a standard deviation of 0.2.

(i) Determine suitable values for c and n. **[9]**

(ii) Without doing any further calculations, sketch the power function for this test. **[3]**

(b) One part of the manufacturer's quality control procedure involves studying how the power supplies perform after a lengthy period of usage. A random sample of 10 power supplies are used for a year and then returned to the manufacturer for testing. A control set of 12 power supplies is chosen randomly and kept unused for a year. Each of these power supplies then has its output voltage measured. The data, ordered by voltage, are as follows.

Used power supplies	14.50	14.67	14.71	14.93	14.99	15.00	15.02	15.05	15.16	15.18		
Unused power supplies	14.77	14.81	14.92	14.94	15.04	15.07	15.10	15.11	15.14	15.26	15.28	15.39

Because of the differing conditions under which some of the power supplies have been used, it is not thought safe to assume underlying Normality.

(i) Carry out a suitable test, at the 5% level of significance, to investigate whether or not usage affects the median output voltage of the power supplies. **[9]**

(ii) What assumption about the variability of voltages is made in the test in part **(i)**? Discuss briefly (but without carrying out any calculations) whether or not this assumption is reasonable. **[3]**

Option 4: Design and Analysis of Experiments

4 **(a)** What is randomised block design and what is its purpose? Describe an experimental situation where randomised block design is appropriate. You should identify the main effect and the blocking factor in your description. **[5]**

(b) What are the distributional assumptions required to conduct an ANOVA test? **[5]**

(c) A sample of 15 people take part in a medical trial to compare 3 different drugs for controlling blood pressure. The sample is split into 3 groups, with each group taking a different drug. Each group contains 5 people. The decrease in the blood pressure of each person is measured (in arbitrary units).

A partially completed ANOVA table is shown below.

Source of variation	**Sum of squares**	**Degrees of freedom**
Between treatments	x	a
Residual		b
Total	1000	

(i) State the standard null and alternative hypotheses for ANOVA, stating the meaning of any symbols used. **[2]**

(ii) Write down the values of a and b. **[2]**

(iii) Find the minimum value of x which results in rejection of the null hypothesis using a 5% significance level. **[10]**

END OF QUESTION PAPER

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Centre number						Candidate number				

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Turn over

1 (i)

1 (ii)

1 (iii)	
1 (iv)	

1 (v)	
1 (vi)	

2 (i)	

2 (ii)	
2 (iii)	

2 (iv)	
3 (a)(i)	
	(answer space continued on next page)

3 (a)(i)	**(continued)**
3 (a)(ii)	
3 (b)(i)	
	(answer space continued on next page)

3(b)(i)	**(continued)**
3(b)(ii)	

4 (a)	

4 (b)	
4 (c)(i)	
4 (c)(ii)	

4 (c)(iii)	

GCE

Mathematics (MEI)

Unit **4769**: Statistics 4

Advanced GCE

Mark Scheme for June 2018

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	
Other abbreviations in mark scheme	**Meaning**
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Statistics strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise.

Candidates are expected to give numerical answers to an appropriate degree of accuracy. 3 significant figures may often be the norm for this, but this always needs to be considered in the context of the problem in hand. For example, in quoting probabilities from Normal tables, we generally expect *some* evidence of interpolation and so quotation to 4 decimal places will often be appropriate. But even this does not always apply – quotations of the standard critical points for significance tests such as 1.96, 1.645, 2.576 (maybe even 2.58 – but not 2.57) will commonly suffice, especially if the calculated value of a test statistic is nowhere near any of these values. Sensible discretion *must* be exercised in such cases.

Discretion must also be exercised in the case of small variations in the degree of accuracy to which an answer is given. For example, if 3 significant figures are expected (either because of an explicit instruction or because the general context of a problem demands it) but only 2 are given, loss of an accuracy ("A") mark is likely to be appropriate; but if 4 significant figures are given, this should not normally be penalised. Likewise, answers which are slightly deviant from what is expected in a very minor manner (for example a Normal probability given, after an attempt at interpolation, as 0.6418 whereas 0.6417 was expected) should not be penalised. However, answers which are *grossly* over- or under-specified should normally result in the loss of a mark. This includes cases such as, for example, insistence that the value of a test statistic is (say) 2.128888446667 merely because that is the value that happened to come off the candidate's calculator. Note that this applies to answers that are given as final stages of calculations; intermediate working should usually be carried out, and quoted, to a greater degree of accuracy to avoid the danger of premature approximation.

The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h Genuine misreading (of numbers or symbols, occasionally even of text) occurs. If this results in the object and/or difficulty of the question being considerably changed, it is likely that all the marks for that question, or section of the question, will be lost. However, misreads are often such that the object and/or difficulty remain substantially unaltered; these cases are considered below.

The simple rule is that *all* method ("M") marks [and of course all independent ("B") marks] remain accessible but at least some accuracy ("A") marks do not. It is difficult to legislate in an overall sense beyond this global statement because misreads, even when the object and/or difficulty remains unchanged, can vary greatly in their effects. For example, a misread of 1.02 as 10.2 (perhaps as a quoted value of a sample mean) may well be catastrophic; whereas a misread of 1.6748 as 1.6746 may have so slight an effect as to be almost unnoticeable in the candidate's work.

A misread should normally attract *some* penalty, though this would often be only 1 mark and should rarely if ever be more than 2. Commonly in sections of questions where there is a numerical answer either at the end of the section or to be obtained and commented on (eg the value of a test statistic), this answer will have an "A" mark that may actually be designated as "cao" [correct answer only]. This should be interpreted *strictly* – if the misread has led to failure to obtain this value, then this "A" mark must be withheld even if all method marks have been earned. It will also often be the case that such a mark is implicitly "cao" even if not explicitly designated as such.

On the other hand, we commonly allow "fresh starts" within a question or part of question. For example, a follow-through of the candidate's value of a test statistic is generally allowed (and often explicitly stated as such within the marking scheme), so that the candidate may exhibit knowledge of how to compare it with a critical value and draw conclusions. Such "fresh starts" are not affected by any earlier misreads.

A misread may be of a symbol rather than a number – for example, an algebraic symbol in a mathematical expression. Such misreads are more likely to bring about a considerable change in the object and/or difficulty of the question; but, if they do not, they should be treated as far as possible in the same way as numerical misreads, *mutatis mutandis*. This also applied to misreads of text, which are fairly rare but can cause major problems in fair marking.

The situation regarding any particular cases that arise while you are marking for which you feel you need detailed guidance should be discussed with your Team Leader.

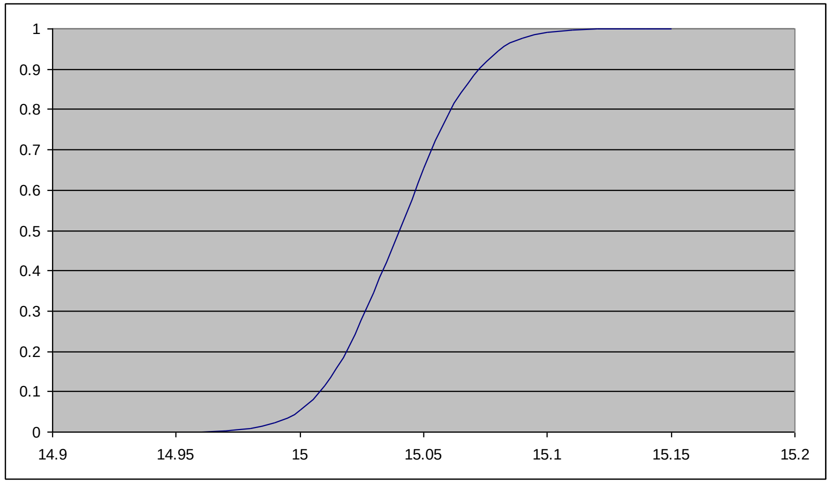
Note that a miscopy of the candidate’s own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1	(i)		$P(T=t)$ $= P(\text{tail on } t^{th}\text{throw}) \times P(1 \text{ tail in } t-1 \text{ throws})$ $= p \times \binom{t-1}{1} p \times (1-p)^{t-2}$ $= p \times (t-1) \times p \times (1-p)^{2}$ $= (t-1)(1-p)^{t-2}p^2$	M1 M1 A1 AG **[3]**	For analysing into final throw and previous situation For bringing in any binomial probability ideas. For correctly simplifying binomial coefficient	
	(ii)		$\frac{dP}{dp} = (t-1)[-(t-2)p^2(1-p)^{t-3} + 2p(1-p)^{t-2}] = 0$ $(t-1)p(1-p)^{t-3}[-(t-2)p + 2(1-p)] = 0$ Since t cannot equal 1, as long as p is neither 0 nor 1 then $2(1-p) = (t-2)p$ $2 - 2p = tp - 2p$ $p = \frac{2}{t}$	M1 A1 M1 A1 **[4]**	M1 setting dP/dp to zero. A1 for accurate differentiation. For getting this far with solving the equation. Reasoning may be implicit	Or use $\mathrm{d}\ln P/\mathrm{d}P=0$
	(iii)		$E\left(\frac{1}{T-1}\right) = \sum_{t=2}^{\infty} \frac{1}{t-1} \times (t-1)(1-p)^{t-2}p^2$ $= p^2 \sum_{2}^{\infty} (1-p)^{t-2}$ $= p^2 \times \frac{1}{1-(1-p)}$ $= p^2 \times \frac{1}{p}$ $= p$	M1 A1 M1 A1 A1	M1 for attempting expectation. A1 for accurate expression, including starting at t=2 Making a link with geometric series.	

Question			Answer	Marks	Guidance	
			Hence $\frac{1}{T-1}$ is an unbiased estimate of p.	AG **[5]**		
	(iv)		$$\mathrm{E}\left(\frac{1}{F}\right) = \sum_{1}^{\infty} \frac{1}{f} pq^{f-1}$$ $$= \frac{p}{q} \sum_{1}^{\infty} \frac{q^f}{f}$$ $$= -\frac{p}{q} \ln(1-q)$$ $$\neq p$$	M1 A1 M1 A1 **[4]**	Manipulating to a form where given formula is applicable. Correct application of formula Must make clear claim at the end.	
	(v)		$$\mathrm{E}(Y) = \sum_{y=1}^{\infty} ypq^{y-1}$$ $$= 1 \times p \times q^{1-1} (+0)$$ $$= p$$ (Therefore Y is an unbiased estimator)	M1 M1 A1 **[3]**	For trying to find E(Y). A general form is not required. Some evidence of $1 \times P(Y = 1)$	
	(vi)		$$\mathrm{E}(Y^2) = \sum_{y=1}^{\infty} y^2 pq^{y-1}$$ $$= p$$ $$\mathrm{Var}(Y) = p - p^2$$ When p=0.5, Var(*Y*)=0.25 and Var(*Z*)=0.0966 So Var(Y)>Var(Z) For unbiased estimators, greater efficiency means lower variance, therefore Z is more efficient.	M1 A1 A1 A1 AG E1 **[5]**	 Both required.	

Question			Answer	Marks	Guidance	
2	(i)		$M(t) = \mathrm{E}(e^{xt}) = \frac{1}{2b}\left(\int_{-\infty}^{0} e^{xt}e^{\frac{x}{b}}\,dx + \int_{0}^{\infty} e^{xt}e^{-\frac{x}{b}}\,dx\right)$	M1 M1 A1	First M1 for correct definition of M(t), second M1 for turning into integral form, A1 for entirely correct expression.	If no definition award for a fully correct expression.
			$= \frac{1}{2b}\left(\int_{-\infty}^{0} e^{x\left(t+\frac{1}{b}\right)}\,dx + \int_{0}^{\infty} e^{x\left(t-\frac{1}{b}\right)}\,dx\right)$	M1	Combining exponents	
			$= \frac{1}{2b}\left(\left[\frac{1}{t+\frac{1}{b}}e^{x\left(t+\frac{1}{b}\right)}\right]_{-\infty}^{0} + \left[\frac{1}{t-\frac{1}{b}}e^{x\left(t-\frac{1}{b}\right)}\right]_{0}^{\infty}\right)$	M1 A1	Attempt at integration, A1 correct, ignore limits	
			If $-1 < bt < 1$ then $t+\frac{1}{b} > 0$ and $t-\frac{1}{b} < 0$ so the integrands vanish at infinity.	E1	Some justification required	
			$M(t) = \frac{1}{2b}\left(\frac{1}{t+\frac{1}{b}} - \frac{1}{t-\frac{1}{b}}\right)$	A1	Correctly found	
			$= \frac{1}{2b}\left(\frac{b}{bt+1} - \frac{b}{bt-1}\right)$			
			$= \frac{1}{2b}\left(\frac{b^2t-b-b^2t-b}{(bt+1)(bt-1)}\right)$	A1	For correct addition of fractions. cao	
			$= \frac{1}{2b}\frac{-2b}{b^2t^2-1}$			
			$= \frac{1}{1-b^2t^2}$	AG		
				[9]		
	(ii)		M'(X)= $2b^2t/(1-b^2t^2)$	M1	**Or** series expansion	
			M''(X) = $2b^2/(1-b^2t^2) + 8b^4t^2/(1-b^2t^2)^3$	M1	Picking out correct term for $E(X^2)$	
				A1	$\mathrm{E}(X^2) = 2b^2$	
			So $\mathrm{E}(X) = 0$	B1		
			$\mathrm{Var}(X) = \mathrm{E}(X^2) - [\mathrm{E}(X)]^2$ or $M''(0) - [M'(0)]^2$	M1		
			$\mathrm{Var}(X) = 2b^2$	A1		
				[6]		

Question			Answer	Marks	Guidance	
	(iii)		Let $W = -Y$. $$M_W(t) = M_Y(-t) = \frac{1}{1+bt} \quad (\text{for } bt > -1)$$	M1A1	M1 for any attempt at using the linear transformation result.	Here, and below, ignore absence or incorrect domains of the MGFs.
			If $D = Y + Ws$ $$M_D = \frac{1}{1-bt} \times \frac{1}{1+bt} \quad (\text{for } \lvert bt \rvert < 1)$$ $$= \frac{1}{1-b^2t^2}$$	M1 A1	For using convolution theorem	
			Since moment generating functions uniquely specify a distribution, this is has the same distribution as X.	E1 **[5]**		
	(iv)		If T is the difference between waiting times then T follows a the same distribution as X with b=4.	M1		
			$$\mathrm{P}(\lvert T \rvert > 0.1) = 2\mathrm{P}(T > 0.1) = 2 \times \int_{0.1}^{\infty} \frac{1}{8} e^{-\frac{x}{4}} \mathrm{d}x$$	M1	With correct limits	
			$$= \left[-e^{-\frac{x}{4}}\right]_{0.1}^{\infty}$$	M1	Correct integration	
			$$= e^{-\frac{1}{40}} \approx 0.975$$	A1 **[4]**		

Question			Answer	Marks	Guidance	
3	(a)	(i)	$\bar{X} \sim \mathrm{N}\left(\mu, \frac{0.04}{n}\right)$	M1	May be implied by working	
			$\mathrm{P}(\bar{X} > c \mid \mu = 15.0) = 0.05$ gives $c = 15.0 + 1.645\frac{0.2}{\sqrt{n}}$	M1A1	M1 for 15 +…Allow equation (inequalities are strictly speaking correct)	
			$\mathrm{P}(\bar{X} < c \mid \mu = 15.1) = 0.01$ gives $c = 15.1 - 2.326\frac{0.2}{\sqrt{n}}$	M1A1	M1 for 15.1 - …	
			Hence $0.1 = 3.971\frac{0.2}{\sqrt{n}}$, giving $n = 63.075 \ldots, c = 15.041 \ldots$	M1A1A1	M1 attempt to solve	
			n could be 64 or more, and $c =$ 15.04	B1	Check that such values actually work not required In fact, n > 63.075… is needed, and the actual bounds for c are given by $15 + \frac{0.329}{\sqrt{n}} < c < 15.1 - \frac{0.4652}{\sqrt{n}}$	
				[9]		
		(ii)		G1	Correct shape	
				B1	Through (15.0, 0.05)	
				B1	Through (15.1, 0.99)	
				[3]		

Question			Answer	Marks	Guidance	
	(b)	(i)	H_0: population medians for used and unused power supplies are equal H_1: population medians for used and unused power supplies are not equal	B1 B1	(Or better; e.g. a formulation in terms of a shift in location parameter for underlying distributions)	Award maximum 1 mark here if there is no reference to populations
			Ranks: Used 1 2 3 7 9 10 11 13 18 19 Unused 4 5 6 8 12 14 15 16 17 20 21 22 Sums of ranks: used: 93 unused: 160	M1A1 A1 ft	Or Mann-Whitney 0+0+0+3+4+4+4+5+9+9 or 93 – 10x11/2 =38 Ft their ranks/counts	
			The critical value for a two-tailed 5% test, m = 10, n = 12 is 84 The observed value of 93>84, not in the critical region Accept H_0 I.e. no reason suppose the median voltage is affected by usage	B1 B1 B1 E1 **[9]**	T crit= 84-10x11/2=29 38>29, not in the critical region	
		(ii)	It is assumed that the variability of voltages (in the populations) is the same for used and unused power supplies. (In fact, it is assumed that the underlying distributions are identical except for a possible shift in location parameter.) It is entirely possible that usage affects the variability, But we have no evidence of this and it seems reasonable to carry out the test.	E1 E1 E1 **[3]**	Reward any two sensible points about variability	

Question			Answer	Marks	Guidance	
4	(a)		Randomised block design is the arrangement of experimental units in blocks by a factor which is not of primary interest. It is used to control for this source of variability, producing a more valid (or accurate) result. Clear description of situation where RBD is valid. Identification of blocking factor Identification of main effect	E1 E1 E1 E1 E1 **[5]**		
	(b)		The error / residuals must be Normally distributed with mean zero with the same variance in each group. independent	B1 B1 B1 B1 B1 **[5]**	allow uncorrelated	
	(c)	(i)	$H_0: \mu_1 = \mu_2 = \mu_3$ where μ_i is the population mean decrease in blood pressure associated with being in the i^{th} group / i^{th} drug. H_1: Not all (population) means are equal	B1 B1 **[2]**	Must use μ or state population mean.	
		(ii)	$a = 2$ $b = 12$	B1 B1 **[2]**		
		(iii)	$$MS_G = \frac{x}{2}$$ $$MS_E = \frac{1000 - x}{12}$$ $$F = \frac{x/2}{(1000 - x)/12} = \frac{6x}{1000 - x}$$ Critical value is 3.89 So we need $\frac{6x}{1000-x} = 3.89$ $$6x = 3890 - 3.89x$$ $$9.89x = 3890$$	B1 M1A1 M1A1 M1 A1 M1	 M1 for ratio either way round. M1 for evidence of using correct tables and correct degrees of freedom. Or appropriate inequality.	

Question			Answer	Marks	Guidance	
			$x = 393.3266 \approx 393$ $x > 393$ or 393.2	A1 B1 **[10]**	393 or 393.2 Inequality used	

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The Triangle Building
Shaftesbury Road
Cambridge
CB2 8EA

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Unit level raw mark and UMS grade boundaries June 2018 series

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AS GCE / Advanced GCE / AS GCE Double Award / Advanced GCE Double Award

AS & Advanced GCE Mathematics											
				Max Mark	a	b	c	d	e	u
4721	01	C1 Core mathematics 1 (AS)	Raw	72	61	55	50	45	40	0
			UMS	100	80	70	60	50	40	0
4722	01	C2 Core mathematics 2 (AS)	Raw	72	55	49	43	37	31	0
			UMS	100	80	70	60	50	40	0
4723	01	C3 Core mathematics 3 (A2)	Raw	72	55	48	41	34	28	0
			UMS	100	80	70	60	50	40	0
4724	01	C4 Core mathematics 4 (A2)	Raw	72	54	47	40	34	28	0
			UMS	100	80	70	60	50	40	0
4725	01	FP1 Further pure mathematics 1 (AS)	Raw	72	56	50	45	40	35	0
			UMS	100	80	70	60	50	40	0
4726	01	FP2 Further pure mathematics 2 (A2)	Raw	72	59	53	47	41	35	0
			UMS	100	80	70	60	50	40	0
4727	01	FP3 Further pure mathematics 3 (A2)	Raw	72	47	41	36	31	26	0
			UMS	100	80	70	60	50	40	0
4728	01	M1 Mechanics 1 (AS)	Raw	72	60	51	42	34	26	0
			UMS	100	80	70	60	50	40	0
4729	01	M2 Mechanics 2 (A2)	Raw	72	53	46	39	32	26	0
			UMS	100	80	70	60	50	40	0
4730	01	M3 Mechanics 3 (A2)	Raw	72	50	42	34	27	20	0
			UMS	100	80	70	60	50	40	0
4731	01	M4 Mechanics 4 (A2)	Raw	72	59	53	47	42	37	0
			UMS	100	80	70	60	50	40	0
4732	01	S1 – Probability and statistics 1 (AS)	Raw	72	57	50	43	36	29	0
			UMS	100	80	70	60	50	40	0
4733	01	S2 – Probability and statistics 2 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4734	01	S3 – Probability and statistics 3 (A2)	Raw	72	59	50	41	32	24	0
			UMS	100	80	70	60	50	40	0
4735	01	S4 – Probability and statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4736	01	D1 – Decision mathematics 1 (AS)	Raw	72	55	48	42	36	30	0
			UMS	100	80	70	60	50	40	0
4737	01	D2 – Decision mathematics 2 (A2)	Raw	72	58	53	48	44	40	0
			UMS	100	80	70	60	50	40	0

AS & Advanced GCE Mathematics (MEI)

				Max Mark	a	b	c	d	e	u
4751	01	C1 – Introduction to advanced mathematics (AS)	Raw	72	60	55	50	45	40	0
			UMS	100	80	70	60	50	40	0
4752	01	C2 – Concepts for advanced mathematics (AS)	Raw	72	53	47	41	36	31	0
			UMS	100	80	70	60	50	40	0
4753	01	(C3) Methods for Advanced Mathematics (A2): Written Paper	Raw	72	61	56	51	46	40	0
4753	02	(C3) Methods for Advanced Mathematics (A2): Coursework	Raw	18	15	13	11	9	8	0
4753	82	(C3) Methods for Advanced Mathematics (A2): Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4754	01	C4 – Applications of advanced mathematics (A2)	Raw	90	63	56	49	43	37	0
			UMS	100	80	70	60	50	40	0
4755	01	FP1 – Further concepts for advanced mathematics (AS)	Raw	72	55	51	47	43	40	0
			UMS	100	80	70	60	50	40	0
4756	01	FP2 – Further methods for advanced mathematics (A2)	Raw	72	48	42	36	31	26	0
			UMS	100	80	70	60	50	40	0
4757	01	FP3 – Further applications of advanced mathematics (A2)	Raw	72	63	56	49	42	35	0
			UMS	100	80	70	60	50	40	0
4758	01	(DE) Differential Equations (A2): Written Paper	Raw	72	61	54	48	42	35	0
4758	02	(DE) Differential Equations (A2): Coursework	Raw	18	15	13	11	9	8	0
4758	82	(DE) Differential Equations (A2): Carried Forward Coursework Mark	Raw	18	15	13	11	9	8	0
			UMS	100	80	70	60	50	40	0
4761	01	M1 – Mechanics 1 (AS)	Raw	72	51	44	37	31	25	0
			UMS	100	80	70	60	50	40	0
4762	01	M2 – Mechanics 2 (A2)	Raw	72	59	53	47	41	35	0
			UMS	100	80	70	60	50	40	0
4763	01	M3 – Mechanics 3 (A2)	Raw	72	61	54	48	42	36	0
			UMS	100	80	70	60	50	40	0
4764	01	M4 – Mechanics 4 (A2)	Raw	72	59	51	44	37	30	0
			UMS	100	80	70	60	50	40	0
4766	01	S1 – Statistics 1 (AS)	Raw	72	59	53	47	42	37	0
			UMS	100	80	70	60	50	40	0
4767	01	S2 – Statistics 2 (A2)	Raw	72	54	47	41	35	29	0
			UMS	100	80	70	60	50	40	0
4768	01	S3 – Statistics 3 (A2)	Raw	72	61	54	47	41	35	0
			UMS	100	80	70	60	50	40	0
4769	01	S4 – Statistics 4 (A2)	Raw	72	56	49	42	35	28	0
			UMS	100	80	70	60	50	40	0
4771	01	D1 – Decision mathematics 1 (AS)	Raw	72	50	44	38	32	26	0
			UMS	100	80	70	60	50	40	0
4772	01	D2 – Decision mathematics 2 (A2)	Raw	72	55	51	47	43	39	0
			UMS	100	80	70	60	50	40	0
4773	01	DC – Decision mathematics computation (A2)	Raw	72	46	40	34	29	24	0
			UMS	100	80	70	60	50	40	0
4776	01	(NM) Numerical Methods (AS): Written Paper	Raw	72	57	52	48	44	39	0
4776	02	(NM) Numerical Methods (AS): Coursework	Raw	18	14	12	10	8	7	0
4776	82	(NM) Numerical Methods (AS): Carried Forward Coursework Mark	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
4777	01	NC – Numerical computation (A2)	Raw	72	55	47	39	32	25	0
			UMS	100	80	70	60	50	40	0
4798	01	FPT - Further pure mathematics with technology (A2)	Raw	72	57	49	41	33	26	0
			UMS	100	80	70	60	50	40	0

AS GCE Statistics (MEI)

				Max Mark	a	b	c	d	e	u
G241	01	Statistics 1 MEI	Raw	72	No entry in June 2018					
			UMS	100	80	70	60	50	40	0
G242	01	Statistics 2 MEI	Raw	72	No entry in June 2018					
			UMS	100	80	70	60	50	40	0
G243	01	Statistics 3 MEI	Raw	72	No entry in June 2018					
			UMS	100	80	70	60	50	40	0

AS GCE Quantitative Methods (MEI)

				Max Mark	a	b	c	d	e	u
G244	01	Introduction to Quantitative Methods (Written Paper)	Raw	72	58	50	43	36	28	0
G244	02	Introduction to Quantitative Methods (Coursework)	Raw	18	14	12	10	8	7	0
			UMS	100	80	70	60	50	40	0
G245	01	Statistics 1	Raw	72	61	55	49	43	37	0
			UMS	100	80	70	60	50	40	0
G246	01	Decision Mathematics 1	Raw	72	50	44	38	32	26	0
			UMS	100	80	70	60	50	40	0

Level 3 Certificate, Level 3 Extended Project and FSMQ raw mark grade boundaries June 2018 series

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Level 3 Certificate Mathematics - Quantitative Methods (MEI)

					Max Mark	a	b	c	d	e	u
G244	A	01	Introduction to Quantitative Methods with Coursework (Written Paper)	Raw	72	58	50	43	36	28	0
G244	A	02	Introduction to Quantitative Methods with Coursework (Coursework)	Raw	18	14	12	10	8	7	0
				UMS	100	80	70	60	50	40	0
				Overall	90	72	62	53	44	35	0

Level 3 Certificate Mathematics - Quantitative Reasoning (MEI)

				Max Mark	a	b	c	d	e	u
H866	01	Introduction to quantitative reasoning	Raw	72	56	49	42	35	28	0
H866	02	Critical maths	Raw	60	44	39	34	29	24	0
		*To create the overall boundaries, component 02 is weighted to give marks out of 72	Overall	144	109	96	83	70	57	0

Level 3 Certificate Mathematics - Quantitative Problem Solving (MEI)

				Max Mark	a	b	c	d	e	u
H867	01	Introduction to quantitative reasoning	Raw	72	56	49	42	35	28	0
H867	02	Statistical problem solving	Raw	60	40	36	32	28	24	0
		*To create the overall boundaries, component 02 is weighted to give marks out of 72	Overall	144	104	92	80	69	57	0

Advanced Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6993	01	Additional Mathematics	Raw	100	56	50	44	38	33	0

Intermediate Free Standing Mathematics Qualification (FSMQ)

				Max Mark	a	b	c	d	e	u
6989	01	Foundations of Advanced Mathematics (MEI)	Raw	40	35	30	25	20	16	0